

Crop planting strategy based on mathematical programming model

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Abstract

This study uses a linear programming model to describe the optimal planting strategy for plants. First, the data is processed to obtain the price, per-acre yield, and cost matrix of different crops in different plots. For the price of crops, this study believes that it fluctuates around a certain mean, so the average sales price in 2023 is taken as the price of agricultural products in the future. There are two cases. A mathematical programming model is established with the goal of maximizing planting benefits to describe the optimal planting strategy between 2024 and 2030. Considering the uncertainty of price, sales volume, pre-acre yield, and planting cost, the linear programming model is combined with Monte Carlo simulation to establish a planning model to describe the optimal planting strategy between 2024 and 2030.

Keywords

Mathematical Programming Model; Planting Strategy; Monte Carlo Simulation

1. Introduction

Making full use of limited arable land resources and developing the organic planting industry according to local natural conditions and soil characteristics has great practical significance for promoting the sustainable development of the rural economy. Carefully selecting suitable crop varieties and optimizing planting strategies can not only help simplify field management and improve agricultural production efficiency, but also effectively reduce planting risks caused by various unpredictable factors.

Many scholars have conducted research on crop planting strategies. Hang Yanpei[1] and others proposed that crop yields can be increased by rationally allocating crop density and combining it with rational density. Shi Wen[2] proposed a crop planting optimization model based on genetic algorithms to study the crop production plan optimization system. Li Xiaowei[3] and others used big data analysis to study crop planting decisions and used support vector machines to build a model. Tang

Shenyi[4] and others used a two-stage method to solve a multi-objective optimization model, providing theoretical guidance for crop planting strategies. Zhao Wanqi[5], Tian Man[6] and others used linear programming models to explore the problem.

There are usually some restrictions on the planting of crops. Crops cannot be planted continuously on the same piece of land. Because the soil containing legume root fungi is conducive to the growth of other crops, legumes must be planted on each piece of land at least once every three years starting from 2023. Considering the convenience of farming operations and field management, each crop cannot be planted too scattered, and the planting area of each crop on a single piece of land cannot be too small.

2. Data Preprocessing

The data source is the existing arable land, suitable crops, and planting conditions of a village in 2023 provided by Question C[7] of the 2024 China Undergraduate Mathematical Contest in Modeling.

Because the planting area, planting season, per-acre yield, planting cost, and sales price of each crop on different land are different, it is necessary to find the planting area, planting season, per-acre yield, planting cost, and sales price of each crop on each piece of land. For land that can be cultivated for two seasons, set a new land number to represent the first and second seasons of this land, and get a matrix of shape 82*41.

Because the expected sales volume remains stable with the sales volume in 2023, this article considers the expected sales volume to be the total production volume in 2023, that is, the supply and sales balance is maintained in 2023. Below is a bar chart of the total sales volume of each crop in 2023 and a bar chart of the arable area of each plot.

Figure 1. Bar chart of total sales of 41 crops

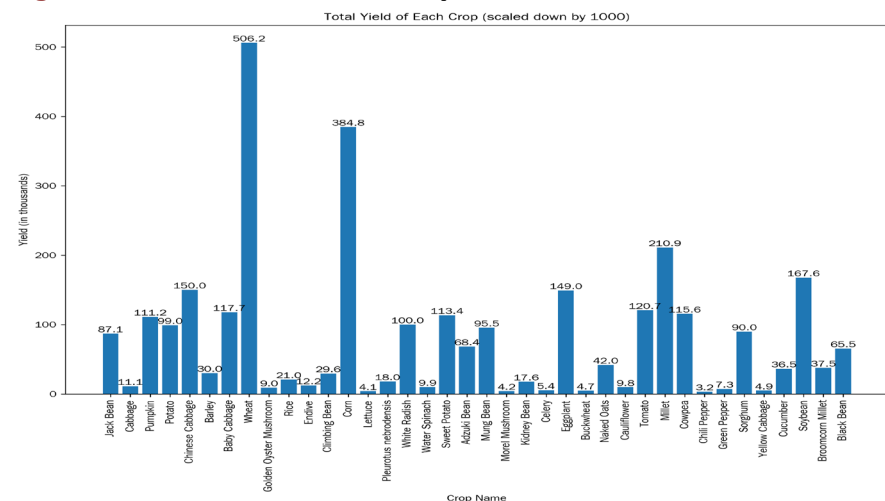
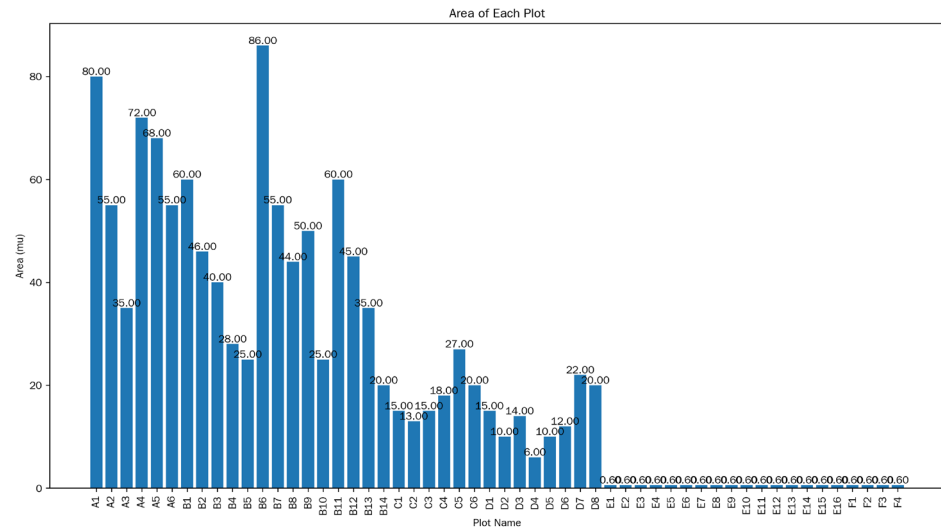


Figure 2. Histogram of arable of each plot

Calculate the area and income for each type of cultivated land. The area of each type of cultivated land is the sum of the areas of plots of the same type. The calculation formula for the income of each type of cultivated land is:

$$A_k = \sum_{i=1}^{n_k} \sum_{j=1}^{41} (m_{ij} \cdot e_{ij} \cdot p_{ij} - m_{ij} \cdot c_{ij}) \quad (1)$$

in:

A_k For the k benefits of cultivated land types;

n_k For the k Total number of plots of cultivated land type;

m_{ij} For the i Plot type planting j Total area planted with crops;

e_{ij} For the i Plot type planting j The yield of crops per acre;

p_{ij} For the i Plot type planting j Unit price of crops grown;

c_{ij} For the i Plot type planting j the cost of growing crops;

Draw a pie chart:

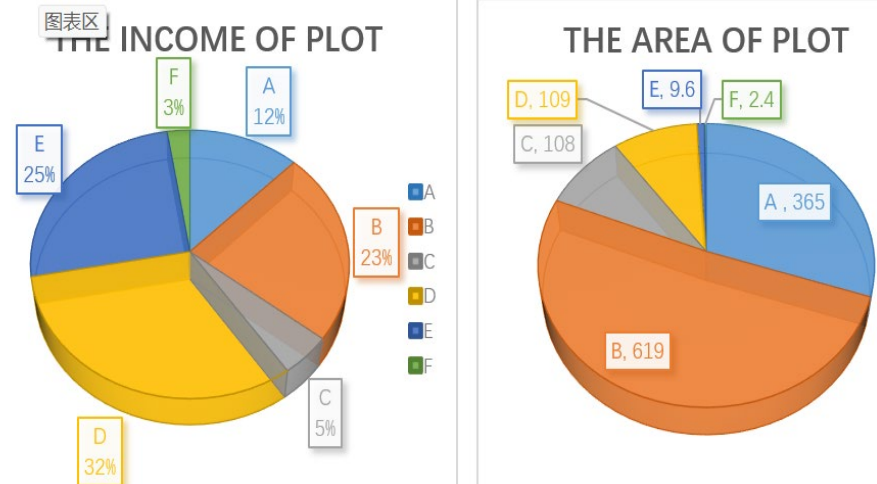


Figure 2. Pie chart of area and income of various types of land

As can be seen from the pie chart, terraced fields account for more than half of the total land area, but the income they obtain is less than a quarter of the total income. Ordinary greenhouses use less than 1% of the total land area, but obtain 25% of the total income. Therefore, when modeling, more consideration should be given to planting in ordinary greenhouses.

3. Planning Model

3.1. Decision variables

Table 1. Symbols and descriptions of decision variables

Symbol	Significance
x_{ijk}	No. k Year i Plot of land j Area under cultivation
P_{ijk}	No. k Year i Plot planting j The price of crops
D_{jk}	No. k Year j Expected sales of the crop
A_i	No. i The area of the block
c_{ijk}	No. k Year i Plot planting j The cost of growing crops
e_{ijk}	No. k Year i Plot planting j Yield of crops per acre

3.2. Model Building

After data processing, the planting area, planting season, per-acre yield, planting cost and sales price of each crop on different land were obtained. Each piece of land that can be planted for two seasons is numbered into the first season and the second season, and is modeled as different land. First, the planting area of each piece of land must be less than the area of this piece of land. For the restrictions on the types of cultivated land for planting various crops, this paper considers using the total area of the crop planted on other cultivated land as 0 to represent it, so that the planting area

of each crop on the cultivated land that cannot be cultivated is 0.

For the restriction of continuous cropping, set a 0-1 variable y_{ijk} , used to record whether each crop is planted on each piece of land, For the constraint that the same crop cannot be cultivated on the same piece of land for two consecutive years, the y_{ij} , If the sum is less than 1, the land that can be planted in two seasons needs to calculate the planting situation for four consecutive seasons.

Similarly, you can use three, consecutive years y_{ij} If the sum is greater than 1, it means that legumes were planted at least once in three years. For land that can be planted twice, six consecutive planting seasons need calculated.

The two cases have the same constraints, In the first case, the part of production that exceeds the sales volume is unsold and wasted, In the second case, the part that exceeds the expected sales volume is sold 50% of the sales price in 2023.

The first scenario: the part of the production that expected the expected sales volume is unsold and cause waste

Objective function: The goal is to maximize crop yields over seven years

$$\max z = \sum_{i=1}^{82} \sum_{j=1}^{41} \sum_{k=1}^7 \left(P_{ijk} \cdot \min \left(\sum_{i=1}^{82} x_{ijk} \cdot e_{ijk}, D_{jk} \right) - c_{ijk} \cdot x_{ijk} \right) \quad (2)$$

0-1 variables:

$$y_{ijk} = \begin{cases} 0, & \text{In the } k\text{-th year, the } j\text{-th crop is not planted on the } i\text{-th piece of land.} \\ 1, & \text{In the } k\text{-th year, the } j\text{-th crop is planted on the } i\text{-th piece of land.} \end{cases} \quad (3)$$

The planting area per plot is less than the area of each plot:

$$\sum_{j=1}^{41} x_{ijk} \leq A_i \quad \forall i = 1, 2, \dots, 82; k = 1, 2, \dots, 7 \quad (4)$$

Restrictions on flat dry land, terraced fields and hillside land:

$$\sum_{i=27}^{82} x_{ijk} = 0 \quad \forall j = 1, 2, \dots, 15; k = 1, 2, \dots, 7 \quad (5)$$

Rice can only be grown for a single season on irrigated land:

$$\sum_{i=1}^{26} x_{i16k} + \sum_{i=35}^{82} x_{i16k} = 0, k = 1, 2, \dots, 7 \quad (6)$$

Restrictions for irrigated land season 1, ordinary greenhouse season 1, smart greenhouse season 1, and season 2:

$$\sum_{i=1}^{26} x_{ijk} + \sum_{i=55}^{78} x_{ijk} = 0, j = 17, 18, \dots, 34 \quad (7)$$

Water Irrigation Season 2 Restrictions:

$$\sum_{i=1}^{54} x_{ijk} + \sum_{i=63}^{82} x_{ijk} = 0, j = 35, 36, 37; k = 1, 2, \dots, 7 \quad (8)$$

General greenhouse season 2 restrictions:

$$\sum_{i=1}^{62} x_{ijk} + \sum_{i=79}^{82} x_{ijk} = 0, j = 38, 39, \dots, 41; k = 1, 2, \dots, 7 \quad (9)$$

You cannot grow the same crop two years in a row:

$$y_{ijk} + y_{ijk+1} \leq 1, i = 1, 2, K, 26; j = 1, 2, K, 41, k = 1, 2, K, 7 \quad (10)$$

$$y_{ijk} + y_{(i+28)jk} + y_{ijk+1} + y_{(i+28)jk+1} \leq 1, i = 27, 28, K, 54, j = 1, 2, K, 41, k = 1, 2, K, 7 \quad (11)$$

Plant soybean crops at least once in three years:

$$y_{ijk} + y_{ijk+1} + y_{ijk+2} \geq 1, i = 1, 2, K, 26; j = 1, 2, K, 5, 17, K, 19; k = 1, 2, K, 7 \quad (12)$$

$$\begin{aligned} y_{ijk} + y_{(i+28)jk} + y_{ijk+1} + y_{(i+28)jk+1} + y_{ijk+2} + y_{(i+28)jk+2} &\geq 1 \\ i = 27, 28, K, 54; j = 1, 2, K, 5, 17, K, 19; k = 1, 2, K, 7 \end{aligned} \quad (13)$$

Non-negativity constraints:

$$x_{ijk} \geq 0, \forall i = 1, 2, K, 82; j = 1, 2, K, 7 \quad (14)$$

Minimum cultivated land area constraint:

$$0.1 \cdot y_{ijk} \leq x_{ijk}, \forall i = 1, 2, K, 82; j = 1, 2, K, 41, k = 1, 2, K, 7 \quad (15)$$

Scenario 2: The portion exceeding the expected sales volume is sold at 50% of the 2023 sales price

Objective function: The goal is to maximize crop yields over seven years

$$\max z = \sum_{i=1}^{82} \sum_{j=1}^{41} \sum_{k=1}^7 (P_{ijk} \cdot \min(Q, D_{jk}) + 0.5P_{ijk} \cdot \max(Q - D_{jk}, 0) - c_{ijk} \cdot x_{ijk}), Q = \sum_{i=1}^{82} x_{ijk} \cdot e_{ij} \quad (16)$$

The constraints are the same as those in the first case and will not be repeated here.

3.3. Planning Model Combined with Monte Carlo Simulation

Taking into account the uncertainties of price, sales volume, per-acre yield and planting costs, the Monte Carlo simulation was used to simulate the rate of change of each data, and then a planning model with random numbers was constructed to describe the optimal planting strategy from 2024 to 2030.

Symbol	Significance	Range variation
r_1	Wheat and corn sales	5%~10%
r_2	Sales of other crops	-5%~5%
α	Change rate of crop yield per mu	-10%~10%
β	Change rate of crop planting cost	5%
γ_1	Vegetable price change rate	5%
γ_2	Price change rate of edible fungi (excluding morels)	-5%~1%
γ_3	Price change rate of morels	-5%

Wheat and corn income:

$$z_1 = \sum_{i=1}^{82} \sum_{j=6}^7 \sum_{k=1}^7 \left(P_{ij1} \cdot \min(Q, R) + 0.5 P_{ij1} \cdot \max(Q - R, 0) - c_{ij1} \cdot (1 + \beta)^{k-1} \cdot x_{ijk} \right) \quad (17)$$

$$\sum_{i=1}^{82} x_{ijk} \cdot e_{ij1} \cdot (1 + \alpha)^{k-1}, R = D_{j1} \cdot (1 + r_1)^{k-1}$$

Income from vegetables:

$$z_2 = \sum_{i=1}^{82} \sum_{j=17}^{34} \sum_{k=1}^7 \left(T \cdot \min(Q, R) + 0.5 P_{ij1} \cdot \max(Q - R, 0) - c_{ij1} \cdot (1 + \beta)^{k-1} \cdot x_{ijk} \right) \quad (18)$$

$$\sum_{i=1}^{82} x_{ijk} \cdot e_{ij1} \cdot (1 + \alpha)^{k-1}, R = D_{j1} \cdot (1 + r_2)^{k-1}, T = P_{ij1} \cdot (1 + \gamma_1)^{k-1}$$

Income from edible fungi (except morels):

$$z_3 = \sum_{i=1}^{82} \sum_{j=38}^{40} \sum_{k=1}^7 \left(T \cdot \min(Q, R) + 0.5 P_{ij1} \cdot \max(Q - R, 0) - c_{ij1} \cdot (1 + \beta)^{k-1} \cdot x_{ijk} \right) \quad (19)$$

$$\sum_{i=1}^{82} x_{ijk} \cdot e_{ij1} \cdot (1 + \alpha)^{k-1}, R = D_{j1} \cdot (1 + r_2)^{k-1}, T = P_{ij1} \cdot (1 + \gamma_2)^{k-1}$$

Morel's income:

$$z_4 = \sum_{i=1}^{82} \sum_{k=1}^7 \left(T \cdot \min(Q, R) + 0.5 P_{ij1} \cdot \max(Q - R, 0) - c_{ij1} \cdot (1 + \beta)^{k-1} \cdot x_{ijk} \right) \quad (20)$$

$$\sum_{i=1}^{82} x_{ijk} \cdot e_{ij1} \cdot (1 + \alpha)^{k-1}, R = D_{j1} \cdot (1 + r_2)^{k-1}, T = P_{ij1} \cdot (1 + \gamma_3)^{k-1}, j = 41$$

Objective function:

$$z = \max E(z_1 + z_2 + z_3 + z_4) \quad (21)$$

The constraints are the same as before and will not be repeated here.

4. Conclusion

Based on the data of price, sales volume, per acre yield and planting cost given by the 2024 National Undergraduate Mathematical Modeling Competition, this paper gives a single-objective programming model for the planting strategy from 2023 to 2030 in two cases. In order to express the impact of various data changes on the planning model, a linear programming model combined with Monte Carlo simulation is introduced. As for the solution of the model, traditional solvers or intelligent algorithms can be used. This paper uses a mathematical programming model to express the planting strategy of crops, provides a theoretical basis for the rational planting of crops, and has broad application potential.

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