Advances in Engineering Research : Possibilities and Challenges
ISSN Print: 3079-5192
ISSN Online: 3079-5206
https://doi.org/10.63313/AERpc.9039

# Dynamic analysis of spur gears considering dynamic stiffness and geometric eccentricity

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**How to cite this paper:** Zheng, Z. H., & Liu, F. H. (2025). Dynamic analysis of spur gears considering dynamic stiffness and geometric eccentricity. Advances in Engineering Research: Possibilities and Challenges, 2(1), 74-80. ISSN Print: 3079-5192; ISSN Online: 3079-5206.
https://doi.org/10.63313/AERpc.9039
**Published: 2025-07-24**



## Abstract

In gear work, the gear transmission system will inevitably have postpartum geometric eccentricity due to the influ-ence of assembly and manufacturing. This phenomenon can cause the center of rotation to not match the center of mass, which further affects the dynamic response of the gear system. In this paper, a dynamic model of the gear shaft based on the transfer method is proposed to consider the geometric eccentricity fault of the gear, and the factor of the meshing stiffness with the velocity is also considered during the gear meshing process. Finally, the dynamic re-sponse under different eccentricity and initial eccentricity angle is discussed.

## Keywords

Geometric eccentricity; dynamic stiffness; eccentricity; initial eccentricity angle; dynamic response

## 1. Imtroduction

With the wide application of gear systems in industry, the dynamic response analysis of gear systems becomes particularly important in improving the stability of the system and reducing the vibration noise of the system. Therefore, it is necessary to establish a more realistic dynamic model.

Geometric eccentricity leads to the variation of gear center distance during the process of meshing resulting in large deviations between the theoretical and actual contact positions. Luo et al. [1] analyzed the effect of gear center distance variation on the time-varying mesh stiffness by analytical method, where the distance variation was caused by the run-out error of the gear singly.

The eccentricity error was evaluated in terms of the static equilibrium position, and the influence of the eccentricity fluctuation on the pure-squeezed journal velocity and reacting force in bearings was considered. Gu and Velax [2] presented a dynamic model to analyze the load sharing, and the results showed that geometric eccentricities have a significant effect on dynamic behaviors. Zhang et al. [3] proposed a three-dimensional dynamic model with geometric eccentricities for a geared rotor system, and the positional changes were analyzed. A nonlinear dynamic model was established by Xiang and Gao [4]. The dynamic responses and the influence of gear eccentricity were discussed.

Gear meshing force is essential as an important excitation in gear systems, and although the influence of eccentric fault on meshing stiffness has been considered in previous studies [5,6,7], the influence of rotational speed on it has not been considered.

## 2. Spur gear system dynamics modeling

### 2.1. Definition of parameters of spur gear system with geometric eccentricity faults

**Table 1.** Gear pairs data.

| Parameter | Pinion/Gear |
|---|---|
| Number of teeth | 27/36 |
| Tooth width (mm) | 10 |
| Module (mm) | 2.5 |
| Modulus of elasticity (G·Pa) | 207 |
| Initial pressure angle $\alpha_0$(°) | 20 |
| Mesh damping ratio $\xi_m$ | 0.25 |
| Backlash(m) | 1E-4 |

### 2.2. Dynamic model with geometric eccentricity

A six-DOF lumped parameter model of a pair of involute spur gear transmission system arranged horizontally is illustrated in Fig. 1.

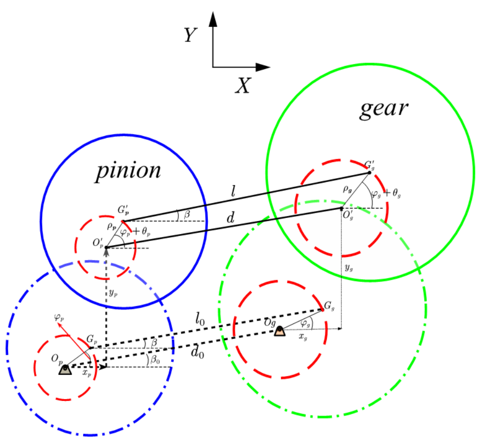


**Fig 1.** Generalized coordinates for the gear pair

In Fig 2, the constant eccentricity of the pinion and gear are $\rho_p$ and$\rho_g$. $O_p$ and $O_g$ represent the rotation centers of the pinion and gear, respectively. $G_p$ and $G_g$ denote two positions of the actual geometric center of the pinion during the rotation process. The motions of the gear set can be defined with six generalized coordinates as they are assumed to be planar motion, namely, two translational coordinates and one rotational coordinate for each gear. $\theta_i$ means the angular displacement of the gear $i$, The vibration displacements of gear $i$ are given by translational coordinates, $x_i$ and $y_i$. $G'_p$ and $G'_g$ are the geometric center at any time after the rotation.

$$\boldsymbol{r}_p = \left(x_p + \rho_p \cos\phi_p\right)\boldsymbol{i} + \left(y_p + \rho_g \sin\phi_p\right)\boldsymbol{j} \tag{1}$$

$$\boldsymbol{r}_g = \left(x_g + \rho_g \cos\phi_g\right)\boldsymbol{i} + \left(y_g + \rho_g \sin\phi_g\right)\boldsymbol{j} \tag{2}$$

where, $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors along the $x$ and $y$ axes, respectively.

The distance $l$ between two centers of geometric can be expressed as:

$$l = \sqrt{(d_0\cos(\beta_0) + X_{G'_g} - X_{G'_p})^2 + (d_0\sin(\beta_0) + Y_{G'_g} - Y_{G'_p})^2} \tag{3}$$

The rotation angle displacements of gear $i$ are given by angular coordinates, $\phi_p$ and $\phi_g$, which can be expressed as follows:

$$\phi_p(t) = \theta_p(t) + \varphi_p, \quad \phi_g(t) = \theta_g(t) + \varphi_g \tag{4}$$

where, $\theta_i$ is the torsional angle displacement superimposed on the rigid-body rotation of gear $i$.

The actual center distance $d$ is given by

$$d = \sqrt{\left(d_0\cos\beta_0 + x_g - x_p\right)^2 + \left(d_0\sin\beta_0 + y_g - y_p\right)^2} \tag{5}$$

In which $d_0 = R_p + R_g$, and $R_p$ and $R_g$ are the pitch circle of the pinion and gear.

The position angle $\beta$ can be expressed as:

$$\beta = \tan^{-1}\frac{d_0\sin\beta_0 + y_g + \rho_g\sin\phi_g - y_p - \rho_p\sin\phi_p}{d_0\cos\beta_0 + x_g + \rho_g\cos\phi_g - x_p - \rho_p\cos\phi_p} \tag{6}$$

The Fig 3 shows a nonlinear dynamic model for the gear pair at any mesh position with eccentricity after motion. The actual pressure angle $\alpha$ is defined as the angle between the $LOA$ and the direction of velocity at the pitch point, and can be expressed as:

$$\alpha = \cos^{-1}\left(\left(R_{bp} + R_{bg}\right)/d\right) \tag{7}$$

where, $R_{bi}$ is the radius of the base circle of gear $i$.

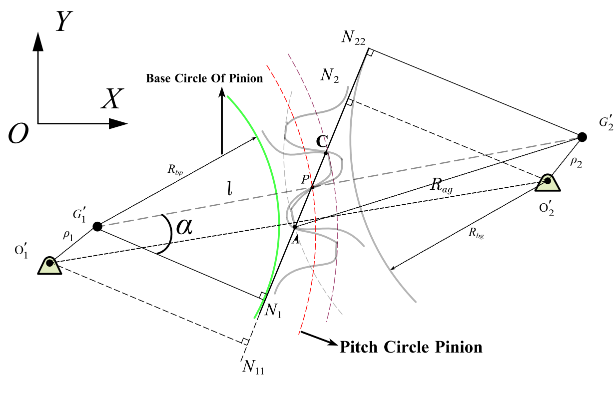


**Fig 2.** Meshing process of a gear pair with eccentricity

According to the gear mesh relationship, the gear mesh deformation (also called $DTE$) between two mating gears along the $LOA$ can be written as:

$$\delta=\left(X_{G_p'}-X_{G_p}-X_{G_g'}+X_{G_g}\right)\sin\left(\alpha-\beta\right)+\left(Y_{G_p'}-Y_{G_p}-Y_{G_g'}+Y_{G_g}\right)\cos\left(\alpha-\beta\right)+R_{bp}\psi_p+R_{bg}\psi_g$$

(8)

The relationship between the coordinates of the geometry center point and the center of rotation can be found from geometric relationships.

$$X_{G_p}=\rho_p\cos\varphi_p\,,\quad Y_{G_p}=\rho_p\sin\varphi_p \tag{9}$$

$$X_{G_g}=\rho_g\cos\varphi_g\,,\quad Y_{G_g}=\rho_g\sin\varphi_g \tag{10}$$

$$X_{G_p'}=x_p+\rho_p\cos\phi_p\,,\quad Y_{G_p'}=y_p+\rho_p\sin\phi_p \tag{11}$$

$$X_{G_g'}=x_g+\rho_g\cos\phi_g\,,\quad Y_{G_g'}=y_g+\rho_g\sin\phi_g \tag{12}$$

The relationship between $\psi_p$ and $\theta_p$, $\psi_g$ and $\theta_g$ can be obtained as:

$$\psi_p=\theta_p+\alpha_0-\alpha\,,\quad \psi_g=\theta_g+\alpha_0-\alpha \tag{13}$$

Eventually, the total gear backlash $b_t$ including the initial backlash $b_0$ can be expressed as:

$$b_t=b_0+\Delta b=b_0+(R_{bp}+R_{bg})(\tan\alpha-\tan\alpha_0+\alpha_0-\alpha) \tag{14}$$

Based on the theory of viscoelasticity, the dynamic mesh force (DMF) consists of elastic force and damping force along the LOA and can be written as:

$$F_{\mathrm{m}}=k_{\mathrm{m}}f\left(\delta,b_t\right)+c_mf_1\left(\delta,b_t\right) \tag{15}$$

where, $k_m$ and $c_m$ are time-varying mesh stiffness and damping, $c_m=2\xi_m\sqrt{k_mJ_1J_2/(J_2R_1^2+J_1R_1^2)}$, $\xi_m$ is the damping ratio (0.03~0.17); $b_t$ is time-varying half gear backlash. $f(\delta,\ b_t)$ and $f_1(\delta,\ b_t)$ are the nonlinear functions for backlash and relative speed respectively and can be expressed as:

$$f\left(\delta,b_t\right)=\begin{cases}\delta-\mathrm{sign}\left(\delta\right)b_t & \left|\delta\right|>b_t\\ 0 & \text{else}\end{cases} \tag{16}$$

$$f_1\left(\delta,\ b_t\right)\begin{cases}\dot{\delta}-\mathrm{sign}\left(\delta\right)\dot{b}_t & \left|\delta\right|>b_t\\ 0 & \text{else}\end{cases} \tag{17}$$

here, superscript $(\cdot)$ denotes derivative with respect to time.

The dynamics equations can be written in matrix form by using Lagrange equation, as follows:

$$\begin{bmatrix}m_p&0&0\\0&m_p&0\\0&0&J_1\end{bmatrix}\begin{bmatrix}\ddot{x}_p\\\ddot{y}_p\\\ddot{\theta}_p\end{bmatrix}+\begin{bmatrix}c_{xp}&0&0\\0&c_{yp}&0\\0&0&c_{\theta p}\end{bmatrix}\begin{bmatrix}\dot{x}_p\\\dot{y}_p\\\dot{\theta}_p\end{bmatrix}+\begin{bmatrix}k_{xp}&0&0\\0&k_{yp}&0\\0&0&k_{\theta p}\end{bmatrix}\begin{bmatrix}x_p\\y_p\\\theta_p\end{bmatrix}+\begin{bmatrix}F_{px}^E+F_{mx_p}^\tau\\F_{py}^E+F_{my_p}^\tau\\F_{p\theta}^E+T_{m_p}^\tau\end{bmatrix}=\begin{bmatrix}0\\-m_pg\\T_p\end{bmatrix}$$

(18)

$$\begin{bmatrix} m_g & 0 & 0 \\ 0 & m_g & 0 \\ 0 & 0 & J_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_g \\ \ddot{y}_g \\ \ddot{\theta}_g \end{bmatrix}+\begin{bmatrix} c_{xg} & 0 & 0 \\ 0 & c_{yg} & 0 \\ 0 & 0 & c_{\theta g} \end{bmatrix}\begin{bmatrix} \dot{x}_g \\ \dot{y}_g \\ \dot{\theta}_g \end{bmatrix}+\begin{bmatrix} k_{xg} & 0 & 0 \\ 0 & k_{yg} & 0 \\ 0 & 0 & k_{\theta g} \end{bmatrix}\begin{bmatrix} x_g \\ y_g \\ \theta_g \end{bmatrix}+\begin{bmatrix} F_{gx}^E + F_{mx_g}^\tau \\ F_{gy}^E + F_{my_{p,g}}^\tau \\ F_{g\theta}^E + T_{m_g}^\tau \end{bmatrix}=\begin{bmatrix} 0 \\ -m_g g \\ T_g \end{bmatrix}$$

(19)

## 3. Calculation of VMS

In one mesh cycle of the gear, there will be a single-tooth mesh region and a double-tooth mesh region. When the two teeth mesh at the same time, it can be seen as a series connection of the V-TVMS of the two pairs of teeth. Hence, the V-TVMS for one mesh cycle can be calculated as:

$$k_{md}\left(\dot{\theta}_p\right)=k_m^1\left(\dot{\theta}_p,\dot{\theta}_g\right)+k_m^2\left(\dot{\theta}_p,\dot{\theta}_g\right) \tag{20}$$

here, the $k_m^1$ and $k_m^2$ are the V-TVMS of single-tooth and double-tooth, respectively. It can be expressed as:

$$k_m^1\left(\dot{\theta}_p,\dot{\theta}_g\right)=\frac{1}{1/k_p^{n_j}\left(\dot{\theta}_p\right)+1/k_g^{n_j}\left(\dot{\theta}_g\right)+1/k_h} \tag{21}$$

$$k_m^2\left(\dot{\theta}_p,\dot{\theta}_g\right)=\begin{cases} \dfrac{1}{1/k_p^{n_n-n_v+n_j}\left(\dot{\theta}_p\right)+1/k_g^{n_n-n_v+n_j}\left(\dot{\theta}_g\right)+1/k_h} & 1\le n_j\le n_v \\ 0 & n_v<n_j\le n_n-n_v+1 \end{cases}$$

(22)

## 4. Dynamic mesh stiffness and dynamics analysis

Then, the gear meshing stiffness in relation to speed can be expressed as shown in the figure

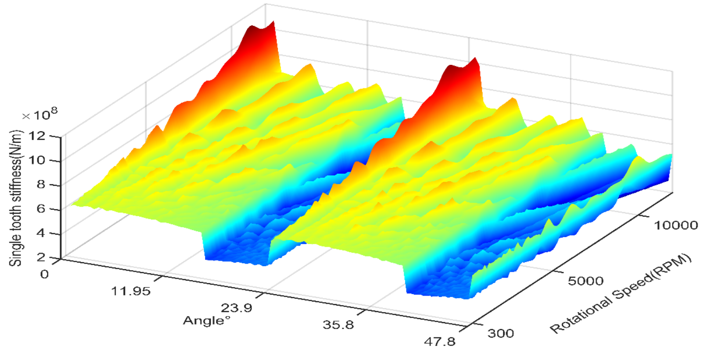


**Fig 3.** 3D plots of comprehensive mesh stiffness with different speeds

Fig 3 shows the 3-D plots of total mesh stiffness with different driving speeds. It is interesting that the driving speed has more effects on the mesh area of double pairs of teeth.

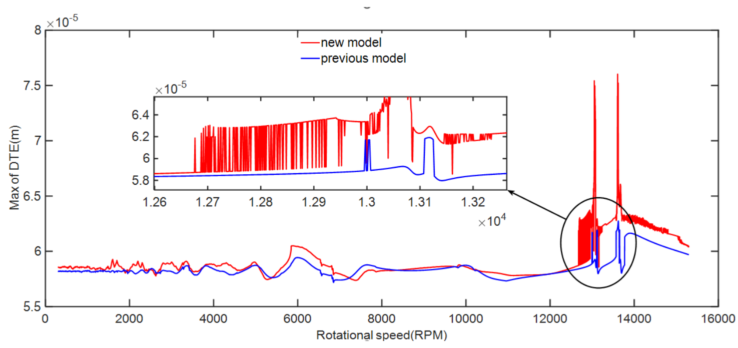


**Fig 4.** Speed sweep of the gear system Red solid line: VMS; Blue solid line: SMS

As shown in fig 4, the abscissa is the driving speed of the gear system, and the ordinate is the maximum value of the dynamic transmission error. The overall trends of the two models are roughly the same before 12,000 rpm. The new model only has a more drastic maximum DTE value with the fluctuation of the speed at medium and low speeds.

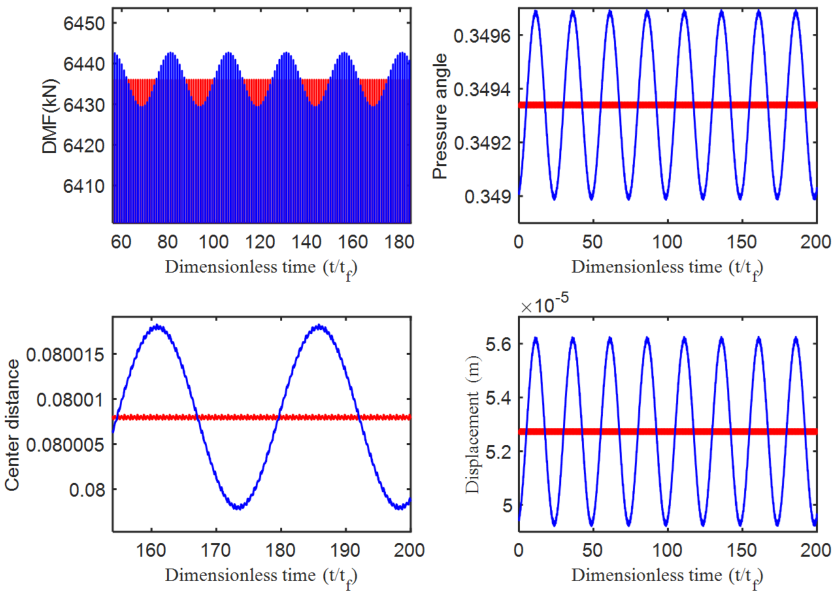


**Fig 5.** Time-domain waveforms in different case: a DMF, b pressure angle, c center distance, d DTE

Due to the time-varying gear eccentricity, parameters such as dynamic meshing force, center distance, and pressure angle also change with time, which affects the dynamic characteristics of the gear system. The states of meshing parameters are divided into two cases. They are the constant parameter (Case 1), and the parameter with eccentricity error (Case 2). When the initial backlash is 50 $\mu m$ and the speed is 6800 rpm, the time-domain waveforms of pressure angle, center distance, dynamic transmission error (DTE), and dynamic mesh force (DMF) are shown in Fig 5. Under the effect of eccentricity error, as can be seen from the a-d of Figure 5, due to the influence of coupling between gear systems, when considering the influence of eccentricity, the various meshing parameters in the gear system will periodically fluctuate up and down between constant parameters, which will directly affect the dynamic characteristics of the gear system.

## 5. Conclusion

In summary, the proposed model is further improved compared with the traditional model, and the influence of gear eccentricity failure on the whole system is studied

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