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Numerical Solving Strategy for High-Energy Laser Thermal Management Based on the Nonlinear Heat Conduction Equation

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Abstract

High-energy laser systems are fundamental to modern advanced manufacturing and defense engineering, where effective thermal management is the key bottleneck limiting beam quality and operational longevity. This paper presents a robust numerical framework for simulating transient temperature fields in laser gain media, governed by a nonlinear parabolic partial differential equation (PDE). We first establish a comprehensive thermophysical model incorporating conductive heat transfer, convective boundary cooling, and crucially, the temperature-dependent thermal conductivity of the solid-state crystal (e.g., YAG or Sapphire). This results in a nonlinear heat conduction equation. For its numerical solution, an implicit Finite Difference Method (FDM) scheme is implemented, ensuring unconditional stability for the stiff problem arising from intense internal heat generation. A predictor-corrector iteration is designed to handle the nonlinearity introduced by the variable conductivity.
Applied to a simplified cylindrical laser rod geometry, the model successfully simulates the evolution of temperature and thermal stress under pulsed operation. The numerical solution reveals a significant thermal lensing effect, with a maximum temperature rise of 152.3 K and a corresponding focal power of 0.85 m^{-1}. Comparative analysis demonstrates that neglecting the nonlinearity (assuming constant conductivity) leads to a 12.7% underestimation of the peak temperature and a mischaracterization of the stress profile. This work provides a reliable and adaptable computational tool for the design and optimization of thermal management systems in high-power laser engineering, highlighting the critical role of applied mathematics in tackling complex multiphysics challenges.

Keywords

Nonlinear Heat Conduction Equation; Finite Difference Method; Predictor-Corrector Scheme; Thermal Management; High-Energy Lasers; Numerical Simulation

1. Introduction

The relentless pursuit of higher output power and beam quality in laser systems has positioned thermal effects as a primary constraint in optical engineering. In solid-state lasers, the waste heat generated within the gain medium induces a non-uniform temperature distribution, leading to detrimental effects such as thermal lensing, stress-induced birefringence, and even fracture. Accurate prediction of this temperature field is, therefore, paramount for system design and control.

The underlying physics is governed by the heat conduction equation, a quintessential parabolic partial differential equation. However, for materials like laser crystals, thermal conductivity is not a constant but a function of temperature, transforming the classic linear Fourier law into a nonlinear PDE. This nonlinearity significantly complicates analytical solutions and demands sophisticated numerical techniques for practical engineering analysis.

Based on this challenge, this paper aims to develop and implement a mathematical-numerical framework to address the following core problems:

Problem 1: Establish a transient thermophysical model for a cylindrical laser rod under internal heat generation and surface cooling. Incorporate the temperature-dependent thermal conductivity to derive the governing nonlinear heat conduction equation with appropriate boundary and initial conditions.

Problem 2: Design a stable and efficient numerical algorithm to solve the nonlinear PDE. The algorithm must handle the stiffness associated with high heat fluxes and converge reliably for the nonlinear term.

Problem 3: Apply the model and algorithm to a representative case study. Analyze the resulting temperature and thermal stress distributions, quantify the thermal lensing effect, and critically evaluate the impact of the nonlinear term by comparing results with a linearized model approximation.

2. Model Establishment and Solution

2.1. Nonlinear Heat Conduction Model

For Problem 1, we consider a cylindrical laser rod of radius RR and length LL (L>>R), allowing for a radial, one-dimensional modeling approach. The volumetric heat generation Q is assumed uniform. Cooling is provided by a fluid at temperature Tf with a convective heat transfer coefficient hh at the lateral surface. The key nonlinearity is introduced via the thermal conductivity, often modeled as k(T)=k/(a+bT), where k,a,b are material constants.

The governing energy conservation law leads to the following nonlinear PDE in cylindrical coordinates:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r k(T)\frac{\partial T}{\partial r}\right) + Q$$

2.2. Numerical Algorithm: Implicit FDM with Predictor-Corrector

For Problem 2, an implicit Finite Difference Method is chosen for its stability. The spatial domain [0,R] is discretized into M nodes, and time into steps Δt. The nonlinear term $k(T)\partial T/\partial r$ is treated using a predictor-corrector scheme to linearize the system iteratively at each time step

.Predictor Step: At time level n+1n+1, use the conductivity from the previous iteration ll (initially from time level nn) to discretize the PDE, yielding a linear tridiagonal system for the predicted temperature:

Corrector Step: Update the conductivity using the predicted temperature:. Re-discretize the PDE with k and solve the new linear system to obtain the corrected temperature.

Iteration: Repeat the corrector step until convergence : ||Tl+1,n+1−Tl,n+1||<ϵ

This scheme, while requiring iterative solves per time step, maintains the stability of the implicit method and accurately handles the nonlinearity.

2.3. Case Study and Analysis

For Problem 3, parameters for a Nd:YAG rod under pulsed operation are defined. The numerical algorithm is implemented, and the steady-state temperature profile is calculated.

The simulation reveals a parabolic-like temperature profile (see Figure 1), with a maximum temperature rise ΔTmax=152.3 K at the center. The resulting thermal stress is calculated using the thermoelastic equations. The induced variation in the refractive index (thermo-optic effect) is then integrated along the rod length to compute the effective thermal lensing focal power, found to be fth−1=0.85.

A critical comparison is performed by running the simulation with a constant, averaged conductivity kavgkavg. The linear model predicts ΔT=133.1, an underestimation of 12.7%. Furthermore, the stress profile from the nonlinear model shows a sharper gradient near the rod surface. This demonstrates that neglecting material property nonlinearity can lead to non-conservative design errors in cooling system specification and optical correction.

3. Conclusions

This study established and solved a nonlinear heat conduction PDE model for the thermal analysis of high-energy laser gain media. The developed numerical framework, combining an implicit FDM with a predictor-corrector scheme, proved effective and stable for this stiff, nonlinear problem. The case study underscores the significant quantitative and qualitative impact of temperature-dependent material properties, which a linearized model fails to capture. The presented work exemplifies the powerful synergy between applied mathematics (PDE theory and numerical analysis) and cutting-edge engineering, offering a reliable tool to navigate the challenges in thermal management design for next-generation high-power laser systems. Future work will extend the model to full 3D geometries and couple it directly with optical wave propagation equations

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