Frontiers in Engineering
ISSN Print: 3104-4298
ISSN Online: 3104-4301
CD ISSN: 3104-431X
https://doi.org/10.63313/FE.9005

Caustic-Based Modulation of Structured Light Fields along Arbitrary Trajectories

YuChen Xu
College of Physics and Electronic Information Engineering, Zhejiang Normal University, Jinhua, China

How to cite this paper: Xu, Y. C. (2026). Caustic-Based Modulation of Structured Light Fields along Arbitrary Trajectories. *Frontiers in Engineering*, 1(2), 28–33. ISSN Print: 3104-4298; ISSN Online: 3104-4301.
https://doi.org/10.63313/FE.9005
Published: 2026-04-08

Abstract

This paper presents an English course report on the caustic-based modulation of structured light fields. Starting from the angular-spectrum description of non-diffracting beams, an inverse design strategy is introduced to map a prescribed transverse intensity contour into an annular spectral phase. In this way, the optical caustic becomes the geometric skeleton that governs both the beam profile and its subsequent propagation. The method enables sharply confined structured beams with arbitrary shapes, while an additional synthesized phase term translates the structure along user-defined trajectories in three-dimensional space. Elliptic and heart-shaped beams are discussed as representative examples, showing that the same framework supports both straight and accelerating propagation. Experimental observations reported in the source material verify the numerical predictions and confirm the robustness of the generated beams. Because these structured non-diffracting fields exhibit strong intensity gradients and structured orbital angular momentum, they are promising for optical trapping, particle transport, beam shaping, wavefront control, and deep imaging. The study demonstrates that inverse caustic engineering offers a flexible and physically intuitive route for designing advanced structured light fields beyond conventional Bessel, Mathieu, Weber, and Airy beam families.

Keywords

Structured Light; Optical Caustics; Non-diffracting Beam; Angular Spectrum; Beam Acceleration; Orbital Angular Momentum

1. Introduction

Non-diffracting beams have attracted sustained attention because they preserve their transverse structure over an extended propagation distance and often exhibit self-healing or self-accelerating behavior. Classical families such as Bessel, Mathieu, Weber, and Airy beams established the theoretical foundation of propagation-invariant optics, but most of them remain tied to fixed beam morphologies or prescribed trajectories. For modern applications in optical trapping, micro-fabrication, wavefront control, and deep imaging, it is desirable to generate beams whose transverse intensity distribution and propagation path can both be customized.

The source material for this report addresses this demand through a caustic-based inverse design method. Instead of deriving a beam solely from special-function solutions of the Helmholtz equation, the method begins with a target optical structure and reconstructs the spectral phase that makes the desired intensity singularities emerge as optical caustics. This viewpoint offers a direct geometric interpretation of structured light formation and expands the design freedom of non-diffracting beams.

The purpose of this paper is to summarize the physical idea, working procedure, representative results, and application potential of the proposed scheme. Special emphasis is placed on how arbitrary transverse structures are encoded, how accelerating trajectories are introduced, and why the resulting beams are useful for particle manipulation and other photonic tasks.

2. Theoretical Background

2.1. Angular-Spectrum Description of Nondiffracting Beams

Under scalar diffraction theory, a monochromatic optical field can be expressed as a superposition of plane waves in the angular-spectrum domain. For a non-diffracting beam, the transverse spectrum is concentrated on a thin annular ring in k-space, which means that all constituent wave vectors share the same transverse magnitude while differing in azimuth. As a result, the field can be written as a one-dimensional angular integral over the ring, and the propagation dynamics are governed primarily by the spectral phase loaded on that ring.

$$U(x,y,z)=\int_0^{2\pi} A(\theta)e^{i[\Phi(\theta)+k_t(x\cos\theta+y\sin\theta)+k_z z]}\,d\theta$$

Here A(theta) denotes the ring amplitude, Phi(theta) is the spectral phase, and the annular support ensures diffraction-resistant propagation. When Phi(theta) is constant, the expression reduces to a Bessel-type beam. Once Phi(theta) varies with azimuth, the beam develops a structured transverse pattern whose geometry is controlled by the phase distribution.

2.2. Inverse Caustic Design

The dominant field contribution comes from stationary-phase points. Each point on the spectral ring emits a ray, and the ray envelope forms an optical caustic. To match a predefined contour f(θ)=(x(θ),y(θ)), the ray must be tangent to the contour:

$$\frac{d\Phi}{d\theta}=-k_t\left(x(\theta)\cos\theta+y(\theta)\sin\theta\right)$$

By integration, the spectral phase is obtained:

$$\Phi(\theta) = -k_t \int_0^{\theta} \left(x(\theta')\cos\theta' + y(\theta')\sin\theta'\right) d\theta'$$

2.3. 3D Trajectory Control

To steer the beam along a 3D trajectory (X(z),Y(z),z), a synthetic translational phase is added to the base phase:

$$\Phi_{\text{total}}(\theta, z_i) = \Phi_0(\theta) + k_t \left[X(z_i)\cos\theta + Y(z_i)\sin\theta\right]$$

By superposing phases for multiple propagation planes, the beam is guided along arbitrary paths (parabolas, cosine curves, helices).

3. Generation of Arbitrarily Structured Accelerating Beams

3.1. Elliptic Structured Beam

An elliptic beam provides a clear illustration of the method. First, an elliptic contour is defined as the target caustic in the transverse plane. The corresponding annular spectral phase is then calculated through the inverse relation between the tangent rays and the contour. Numerical results show that the maximum intensity is concentrated along the prescribed ellipse, while the side-view evolution confirms that the beam remains non-diffracting over a long propagation distance.

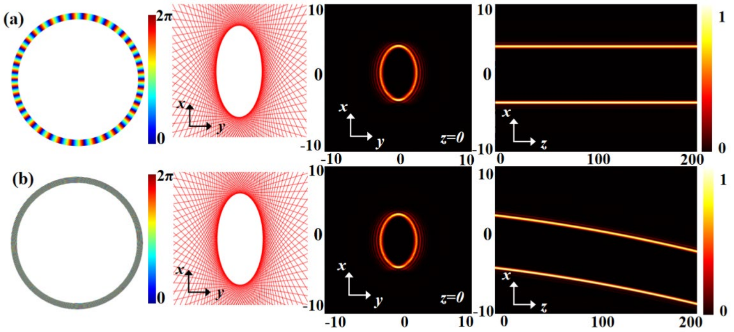

Figure 1. Numerical generation of an elliptic structured non-diffracting beam under straight and parabolic trajectories.

After the structural phase is obtained, a synthesized translation phase is introduced to move the elliptic beam center along a chosen path. In the source results, the same elliptic pattern is successfully forced to follow a parabolic trajectory in the x-z plane. This demonstrates that the method does not merely shape a beam at one plane; it also programs how the structure travels through space.

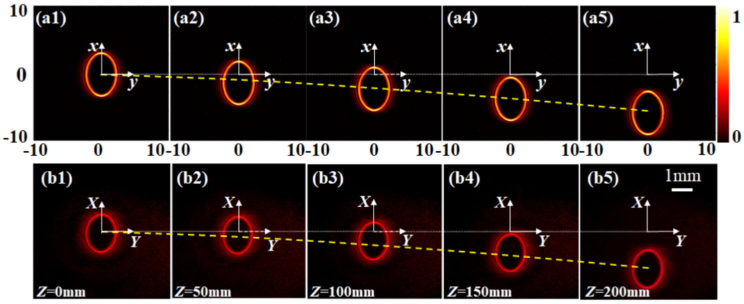

Figure 2. Simulated and experimental propagation of the elliptic beam along a parabolic trajectory.

3.2. Heart-Shaped Structured Beam

To demonstrate generality, the study further considers a heart-shaped beam, which is much more complex than a simple ellipse. The target contour is divided into sub-curves so that separate phase contributions can be derived and then superposed into a total annular phase. The resulting beam preserves the heart-shaped intensity distribution and can simultaneously oscillate along a cosine-like path during propagation.

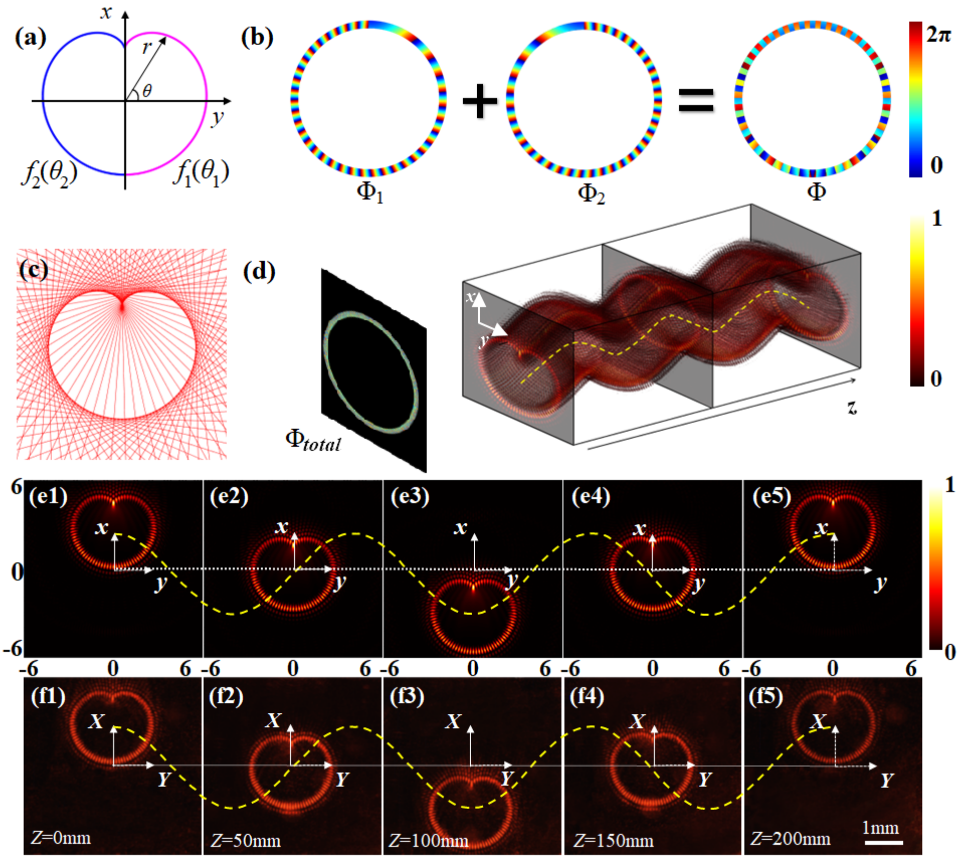

Figure 3. Design and propagation of a heart-shaped non-diffracting beam along a cosine trajectory.

This example is important because it shows that the approach is not restricted to canonical beam families. Any contour that can be parameterized can, in principle, be translated into a caustic skeleton and then into a realizable structured beam. The method therefore offers a scalable route toward highly customized optical fields for application-specific tasks.

4. Experimental Verification and Application Potential

The experimental system described in the source document uses a spatial light modulator to load the designed annular spectral phase onto an incident laser beam. After Fourier transformation by a lens, the structured beam is recorded by a CCD at different propagation planes. The measured intensity distributions agree well with the numerical predictions, confirming the physical validity of the inverse caustic design.

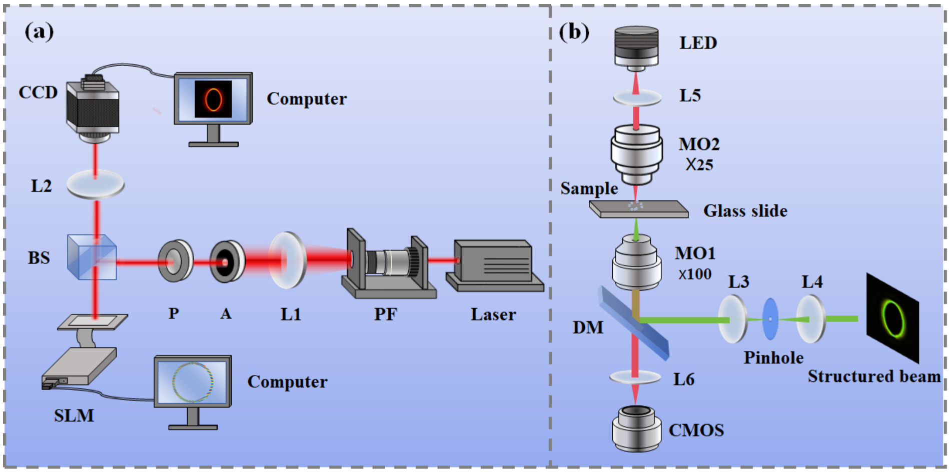

Figure 4. Experimental setup for generating structured non-diffracting beams and applying them to optical manipulation.

A notable feature of these beams is the coexistence of strong transverse intensity gradients and structured phase gradients. This combination leads to nonuniform orbital angular momentum densities, which means that trapped particles can be confined to a specific contour and driven along the same contour with controllable direction and speed. Compared with conventional optical tweezers, such beams offer richer control over particle motion because the trapping path and transport dynamics are encoded directly in the optical field.

Beyond particle manipulation, the proposed scheme is relevant to beam steering, optical micro-machining, complex wavefront synthesis, and deep imaging. Since the design procedure relies on a programmable phase mask rather than a fixed bulk optical element, it is flexible, compact, and compatible with re-configurable photonic systems.

5. Conclusion

In summary, the caustic-based inverse design method provides an intuitive and powerful framework for generating non-diffracting structured light fields with arbitrary transverse shapes and arbitrary propagation trajectories. By linking annular spectral phase engineering to the geometry of optical caustics, the method overcomes the structural rigidity of traditional non-diffracting beam families. Representative elliptic and heart-shaped beams confirm that complex patterns can remain well confined while accelerating along prescribed paths. The experimentally verified results indicate strong potential for optical trapping, particle transport, wavefront control, and imaging. This approach therefore represents a meaningful extension of structured-light engineering in modern optics.

Funding

This research was funded by National Undergraduate Training Program on Innovation and Entrepreneurship，grant number: 202510345027).

References

[1] Durnin, J. (1987). Exact solutions for nondiffracting beams. I. The scalar theory. Journal of the Optical Society of America A, 4(4), 651-654.

[2] Durnin, J., Miceli, J. J., & Eberly, J. H. (1987). Diffraction-free beams. Physical Review Letters, 58(15), 1499-1501.

[3] Berry, M. V., & Balazs, N. L. (1979). Nonspreading wave packets. American Journal of Physics, 47(3), 264-267.

[4] Bandres, M. A. (2008). Accelerating parabolic beams. Optics Letters, 33(15), 1678-1680.

[5] Kaminer, I., Bekenstein, R., Nemirovsky, J., & Segev, M. (2012). Nondiffracting accelerating wave packets of Maxwell's equations. Physical Review Letters, 108(16), 163901.

[6] Greenfield, E., Segev, M., Walasik, W., & Raz, O. (2011). Accelerating light beams along arbitrary convex trajectories. Physical Review Letters, 106(21), 213902.

[7] Chremmos, I., Efremidis, N. K., & Christodoulides, D. N. (2011). Pre-engineered abruptly autofocusing beams. Optics Letters, 36(10), 1890-1892.

[8] McGloin, D., & Dholakia, K. (2005). Bessel beams: Diffraction in a new light. Contemporary Physics, 46(1), 15-28.

[9] Rubinsztein-Dunlop, H., Forbes, A., Berry, M. V., Dennis, M. R., Andrews, D. L., Mansuripur, M., Denz, C., Alpmann, C., Banzer, P., Bauer, T., Karimi, E., Marrucci, L., Padgett, M., Ritsch-Marte, M., Litchinitser, N. M., Bigelow, N. P., Rosales-Guzman, C., Belmonte, A., Torres, J. P., Neely, T. W., Baker, M., Gordon, R., Stilgoe, A. B., Romero, J., White, A. G., Fickler, R., Willner, A. E., Xie, G., McMorran, B., & Weiner, A. M. (2017). Roadmap on structured light. Journal of Optics, 19(1), 013001.