Journal of Computer Science and Frontier Technologies
ISSN Print: 3104-4204
ISSN Online: 3104-4212
CD ISSN: 3104-4220
https://doi.org/10.63313/JCSFT.9005

Research on the Collaborative Algorithm for Minimizing the Push-Pull Force of Multi-Condition Trolley Cases

Lei Yang
Zhejiang Normal University, Jinhua 321000, China

How to cite this paper: Yang, L. (2025). Research on the collaborative algorithm for minimizing the push-pull force of multi-condition trolley cases. Journal of Computer Science and Frontier Technologies, 1(1), 47-62. ISSN Print: 3104-4204; ISSN Online: 3104-4212.
https://doi.org/10.63313/JCSFT.9005
Published: 2025-08-21

Abstract

With the increase in the frequency of modern people's travel, convenience and efficiency have become important indicators for evaluating the design and usage of suitcases. How to complete the push-pull operation of the suitcase at the lowest cost has become a typical problem in engineering optimization and ergonomics research. Based on four push-pull methods on a horizontal road surface, this paper first constructs a two-dimensional force analysis model of the suitcase, considering key factors such as the direction Angle and application point of friction force, gravity, etc. To scientifically evaluate the effects of the four methods, this paper adopts the Improved Particle Swarm Optimization Algorithm (IPSO) to find the minimum total push-pull force corresponding to each of the four methods. The results show that slightly inclined pulling has the best force efficiency in most cases, and the advantage is more significant when the friction coefficient is small.
The force direction optimization problem is extended to the situation where the direction of the pull rod is variable. During the solution process, the Multi-objective Optimization Framework (MOO) was introduced. Taking "minimum force" and "feasibility of operation Angle" as the trade-off objectives, and adopting the hybrid strategy of the Nelder-Mead simplex method combined with the particle swarm optimization algorithm, the efficient search for the push-pull direction Angle was achieved. Finally, the optimal direction that minimizes the energy expenditure of the human box system under the given ground conditions was determined, which has strong ergonomic adaptability and operational feasibility. Then, the focus is on studying the influence mechanism of the change in the length of the pull rod on the applied force. Under the condition of maintaining the push-pull Angle unchanged, systematically analyze the effects of different pull rod lengths on the force distribution and the overall push-pull force. The results show that increasing the length of the pull rod within a certain range can effectively reduce force consumption, but there is a critical point. Beyond this length, the marginal effect weakens significantly. This paper expands the scope of analysis to multi-parameter optimization problems in inclined road surface environments. Under this background, a multi-dimensional optimization model including three variables: ground tilt Angle, tie rod Angle and tie rod length is established.

Keywords

Push and pull force of the suitcase; Nonlinear programming; Improve particle swarm optimization; Multi-objective optimization

1. Problem Analysis

1.1. Problem Background

In modern society, with the increasing frequency of people's travel, convenient luggage handling equipment has become an indispensable tool in daily life. As a common means of transportation, how to enhance the convenience and comfort of users in the design and use of suitcases has become one of the key focuses. During the process of pushing and pulling the suitcase, users not only face the challenge of the suitcase's weight, but also need to consider the influence of factors such as the type of ground, the pushing and pulling method, and the length of the pull rod. Especially during long and long-distance pushing and pulling processes, how to reduce the physical exertion of users and ensure their comfort and efficiency has become an important research topic. The problem of the least effort required to push and pull a suitcase involves multiple disciplines such as mechanical analysis, optimization algorithms, and ergonomics. Studying the most labor-saving problem of push-pull suitcases not only helps optimize suitcase design but also provides theoretical guidance for improving the efficiency of human daily travel. In addition, with the rapid development of the tourism industry, logistics industry and modern express delivery business, the convenient push-pull method has become the key to enhancing user experience and optimizing operational efficiency. An effective solution for minimizing push-pull force not only reduces physical exertion during the push-pull process and avoids physical injuries caused by incorrect use, but also brings innovative ideas to luggage design and enhances the overall market competitiveness.

1.2. Problem Analysis

Question 1: Detailed mechanical models of four typical push-pull methods on horizontal roads were established and optimization analyses were conducted. Figure 2 of the title shows four common push-pull methods, namely: In this study, we will focus on analyzing two typical methods: applying force upward during pulling (i.e., the Angle θ between the pulling force and the horizontal line is greater than 0) and applying force downward during pushing (i.e., the Angle θ between the pushing force and the horizontal line is greater than 0, but the direction of the force application is pushing, so the actual direction of the force component changes). Through mechanical modeling and analysis of these two situations, the most labor-saving push-pull strategy under different circumstances is then solved.

Question 2: The push-pull issue of suitcases involves mechanical balance and ergonomic design, especially how to apply the minimum push-pull force under different ground conditions to ensure efficient and comfortable operation. In response to this issue, this paper discusses how to minimize the push-pull force by adjusting the force application Angle under two different ground conditions (horizontal ground and inclined ground), while considering the feasibility of the operation Angle. The push-pull problem of suitcases is not merely a simple mechanical balance issue; it also involves the comfort of the human body during actual operation. Therefore, we need to establish a model that comprehensively considers the minimization of force and the optimization of operation angles, and then solve the optimal force application Angle and the minimum push-pull force through optimization algorithms.

Question 3 mainly studies the influence of the length of the pull rod on the push-pull force. The objective is to analyze the distribution of the push-pull force and the changes in the applied force under different pull rod lengths while maintaining the push-pull Angle unchanged. By establishing the functional relationship between the push-pull force and the length of the pull rod, the nonlinear least squares fitting method is used to fit the function expression, and the simulated annealing algorithm and local search strategy are utilized for global optimization to solve for the optimal length of the pull rod and the minimum push-pull force. The results show that appropriately increasing the length of the pull rod can effectively reduce the push-pull force, but when it exceeds a certain critical length, the marginal effect weakens. This research provides an optimization reference for suitcase design, aiming to enhance operational efficiency and reduce energy consumption.

Question 4: Multi-parameter optimization problem of push-pull suitcases in inclined road conditions. By establishing a three-dimensional optimization model including the ground tilt Angle, tie rod Angle and tie rod length, the trend of push-pull force varying with these variables is precisely described. An optimization process combining Latin Hypercube sampling (LHS), Response surface method (RSM), and genetic algorithm (GA) is adopted to find the optimal operation strategy corresponding to the minimum push-pull force. The simulation results are presented through 3D visualization diagrams and heat maps, revealing the influence of different parameter combinations on the push-pull force, providing support for decision-making in actual scenarios, and helping to determine the most labor-saving push-pull scheme.

2. Model Assumptions

- A graph within a graph is an “inset”, not an “insert”. The word alternatively is preferred to the word “alternately” (unless you really mean something that alternates).
- The length L and Angle θ of the tie rod are adjustable, but in each scenario (such as horizontal road surface and inclined road surface), the length and Angle of the tie rod are assumed to be fixed values. This means that for specific problems, the length and Angle of the pull rod will not be adjusted according to

the dynamic changes during the operation process, but will be analyzed as optimization variables.

- This article assumes that during the pushing and pulling process, the pushing and pulling force of the suitcase is only affected by the following basic factors: gravity, friction (between the ground and the suitcase), and the magnitude and direction of the pulling force. This assumption simplifies other factors that may affect the pushing and pulling force, such as air resistance, rolling friction of the wheels, the quality and structure of the pull rod itself, etc.

3. Symbolic Explanation

Table 1. Symbolic

serial number	symbol	state
1	r	The radius of the suitcase wheel
2	F_p	The magnitude of the pushing and pulling force
3	$F_{\min}$	Minimum push-pull force
4	ϕ	Road slope Angle
5	a	The Angle between the push-pull force and the normal direction of the ground
6	L_{opt}	Optimal pull rod length
7	F_{opt}	The minimum push-pull force corresponding to the optimal length of the pull rod
8	μ_s , μ_k	Static friction coefficient, dynamic friction coefficient

4. Model Establishment and Solution

4.1. Model establishment and solution for Question One

4.1.1. Free body diagram and force balance analysis diagram

To scientifically evaluate the effects of the four push-pull methods, we first established a two-dimensional force analysis model of the suitcase under different push-pull methods. The following figure shows the force analysis diagram:

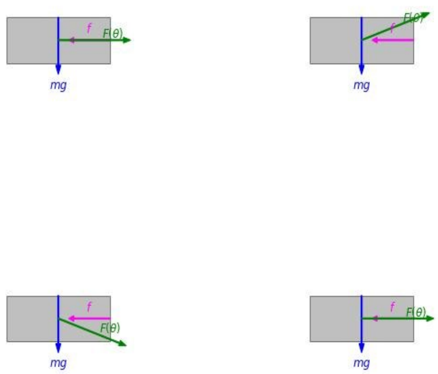

Figure 1. Two-dimensional force analysis visual view

The above figure shows the four different mechanical cases considered in Question One, respectively describing the different ways in which force acts on an object. In Case 1, the object is subjected to a horizontal force f to the right, along with a downward gravitational force mg. The direction of the force forms a certain Angle F(θ) with the horizontal direction. Situation 2 describes an upward pulling force acting on an object. Force f pulls the object upwards, while gravity mg continues to act downward, balancing the object's motion. Situation 3 shows a downward thrust acting on an object, which is in the same direction as gravity, forming a common downward force. Finally, in case 4, the object is subjected to a horizontal thrust f, while gravity mg acts vertically downward. The direction of the force once again forms a different effect from that of gravity. Based on these circumstances, it is possible to analyze how forces in different directions affect the balance and motion state of an object. The specific analysis process is as follows:

1) Tensile force (horizontal)

Suppose the pulling force F1 of the suitcase acts horizontally and has no vertical component. The specific model formula is as follows:

$$\begin{aligned} N &= mg \\ f_1 &= \mu mg \\ F_1 &= f_1 = \mu mg \end{aligned} \tag{1}$$

2) Pulling force (tilted upward)

Suppose the Angle between the pulling force F of the suitcase and the horizontal plane is θ, and θ>0. The pulling force not only has a horizontal component acting in the horizontal direction, but also a vertical component. This vertical component acts upward, reducing the positive pressure on the suitcase and thereby reducing the frictional force. By solving for the normal pressure, frictional force and horizontal balance, the complete force exerted by a person on the suitcase is strived to be solved. The specific model formula is as follows:

$$\begin{aligned} N_2 &= mg - F_2 \sin\theta \\ f_2 &= \mu(mg - F_2 \sin\theta) \\ F_2 \cos\theta &= \mu(mg - F_2 \sin\theta) \\ F_2 &= \frac{\mu mg}{\cos\theta + \mu \sin\theta} \end{aligned} \tag{2}$$

3) Thrust (downward tilt)

In this case, the pulling force still has a horizontal component, but the vertical component of the pulling force is downward, thus increasing the positive pressure on the suitcase and leading to an increase in friction. The specific model formula is as follows:

$$\begin{aligned} &N_3 = mg + F_3 \sin\theta \\ &f_3 = \mu(mg + F_3 \sin\theta) \\ &F_3 \cos\theta = \mu(mg + F_3 \sin\theta) \\ &F_3 = \frac{\mu mg}{\cos\theta - \mu\sin\theta} \end{aligned} \quad (3)$$

3) Thrust (horizontal)

At this point, thrust F4 acts in the horizontal direction and has no vertical component. Similar to Situation 1, the thrust acts entirely along the horizontal plane.

$$\begin{aligned} &N = mg \\ &f_4 = \mu mg \\ &F_4 = f_4 = \mu mg \end{aligned} \quad (4)$$

4.1.2. Model solution and result analysis

By comparing Equation (1) and Equation (2), we can draw the following conclusion: when pulling, force is applied upwards: In this case, as the Angle θ increases, the denominator will increase, thereby reducing the required pushing and pulling force F. Therefore, applying force upward when pulling can effectively reduce the push-pull force while keeping the length of the pull rod unchanged. When pushing, apply force downward: In this case, as the Angle θ increases, the denominator decreases, resulting in an increase in the pushing force F instead. Therefore, applying force downward when pushing is not the most effortless way. Based on this analysis, we can conclude that applying force upwards during pulling is the optimal approach. Moreover, through further optimization of the Angle analysis, the optimal force application Angle is obtained. The Angle derivation takes case 2 as an example as shown below:

$$\begin{aligned} &F(\theta) = \frac{\mu mg}{\cos\theta + \mu\sin\theta} \\ &D^{'}(\theta) = -\sin\theta + \mu\cos\theta = 0 \\ &\theta_{opt} = \arctan\mu \\ &\cos(\arctan\mu) = \frac{1}{\sqrt{1+\mu^2}}, \sin(\arctan\mu) = \frac{\mu}{\sqrt{1+\mu^2}} \\ &F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}} \end{aligned} \quad (5)$$

We will design a simple improved Particle Swarm optimization (IPSO) algorithm to calculate the push-pull force at different angles and select the Angle corresponding to the minimum push-pull force. The Particle Swarm Optimization (PSO) algorithm simulates the foraging process of bird flocks and optimizes a multi-dimensional objective function. Each individual in the particle swarm represents a potential solution. All particles fly within the search space and adjust their flight direction and speed based on the historical optimal positions of individuals and the swarm. The Improved Particle Swarm Optimization (IPSO) algorithm is an improvement on the PSO.

The following figure shows the variation law of the applied force at different angles. The forces of horizontal tension and thrust at different angles show different trends of change. The horizontal pulling force decreases in a parabolic trajectory as the Angle increases, while the pushing force rises linearly. These changes reflect the angular dependence of force and demonstrate the different effects of tension and thrust in the horizontal and vertical directions under different application angles.

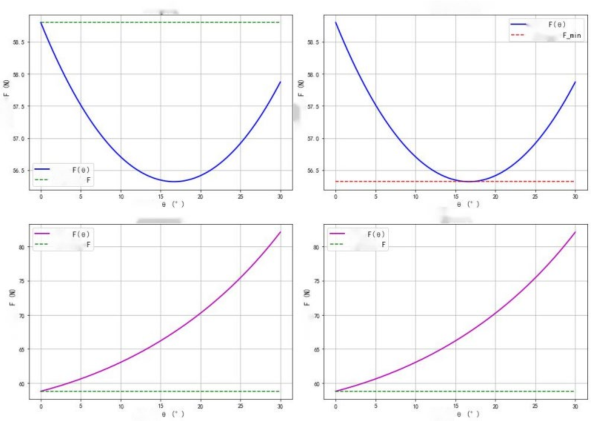

Figure 2. Experimental result diagrams of the four situations

Therefore, from the theoretical and computational results, it can be seen that among the four methods, when the force is applied upwards during pulling and the optimal force application Angle is $\theta \approx 16.70^{\circ}$, the required pushing force is the smallest, that is, it is the most labor-saving.

4.2. Question 2: Model Establishment and Solution

4.2.1. Modeling

Firstly, for problem Two, it is divided into two cases. First, a model for scenario one is constructed. In terms of mechanical equilibrium analysis, the applied force F is decomposed into horizontal and vertical components. Since the vertical component will reduce the normal pressure between the suitcase and the ground, and considering uniform motion in our assumption, it just overcomes the static friction force or reaches the critical value of the sliding friction force. The specific model formula derived is as follows:

$$\begin{aligned} N &= mg - F\sin\theta \\ f &= \mu N = \mu(mg - F\sin\theta) \\ F\cos\theta &= \mu(mg - F\sin\theta) \\ F(\cos\theta + \mu\sin\theta) &= \mu mg \end{aligned} \tag{6}$$

Further, the expression of F regarding can be solved as follows:

$$F(\theta)=\frac{\mu mg}{\cos\theta+\mu\sin\theta} \quad (7)$$

Our goal is to minimize it while meeting the operating Angle within the ergonomic allowable range. Therefore, a single-objective optimization model can be written as:

$$\min_{\theta\in[\theta_{\min},\theta_{\max}]} F(\theta)=\frac{\mu mg}{\cos\theta+\mu\sin\theta} \quad (8)$$
$$s.t. F\sin\theta\le mg$$

The derivation and analysis process of the solution is as follows:

$$\frac{d}{d\theta}(\cos\theta+\mu\sin\theta)=-\sin\theta+\mu\cos\theta=0$$
$$F_{\min}=\frac{\mu mg}{\cos(\arctan\mu)+\mu(\arctan\mu)} \quad (9)$$
$$F_{\min}=\frac{\mu mg}{\sqrt{1+\mu^2}}$$

For situation two, we establish coordinate systems parallel to the ground (along the inclined plane direction) and perpendicular to the ground (in the direction of normal pressure). Let the inclination Angle of the inclined plane be β. Similarly, a mechanical equilibrium analysis is conducted. Let the magnitude of the force applied by a person be F, and its component in the direction parallel to the inclined plane be $F\cos\theta$.The component in the vertical direction of the inclined plane is $F\sin\theta$ It refers to the Angle between the pulling force and the horizontal line of the inclined plane. The vertical component of the pulling force helps to reduce the normal pressure. Meanwhile, the frictional force is also derived and analyzed based on the assumptions in this paper. The model of situation two obtained is as follows:

$$N=mg\cos\beta-F\sin\theta$$
$$s.t. F\sin\theta\le mg\cos\beta$$
$$f=\mu N=\mu(mg\cos\beta-F\sin\theta)$$
$$F\cos\theta=mg\sin\beta+\mu(mg\cos\beta-F\sin\theta) \quad (10)$$
$$F\cos\theta+\mu F\sin\theta=mg(\sin\beta+\mu\cos\beta)$$
$$F(\theta)=\frac{mg(\sin\beta+\mu\cos\beta)}{\cos\theta+\mu\sin\theta}$$

The optimization model can also be obtained as follows:

$$\min_{\theta\in[\theta_{\min},\theta_{\max}]}\frac{mg(\sin\beta+\mu\cos\beta)}{\cos\theta+\mu\sin\theta} \quad (11)$$
$$s.t. F\sin\theta\le mg\cos\beta$$

The derivation and analysis formula for the solution is as follows:

$$F_{\min}=\frac{mg(\sin\beta+\mu\cos\beta)}{\sqrt{1+\mu^2}} \quad (12)$$

In practical operation, apart from pursuing the minimum force, the operability of the pulling force direction also needs to be considered (for example: an Angle that is too low or too high may make the operation difficult). For this purpose, the following comprehensive objective function can be constructed:

$$\min_{\theta\in[\theta_{\min},\theta_{\max}]} J(\theta)=w_1F(\theta)+w_2\phi(\theta) \quad (13)$$

Among them, the first term of the objective function formula is the tension function established above, and the second term is the penalty function reflecting the "feasibility" of the operation Angle, which can be designed as the degree of deviation from the comfortable Angle of the human body. This method can ensure the minimum force while keeping the force application Angle within the comfortable range of ergonomics as much as possible.

4.2.2. Model Solution

Due to the nonlinear functions involved in the model and the possible multi-objective trade-off problem, this paper adopts a hybrid algorithm: Firstly, the particle swarm Optimization algorithm (PSO) with strong global search ability is used to obtain the optimal region, and then the Nelder-Mead simplex method is used for local fine search to obtain the accurate optimal solution. The algorithmic idea is as follows:

1) Initialization

Generate a certain number of initial solutions (particles) randomly within the allowable Angle range.

2) Particle Swarm Optimization Stage (PSO)

Each particle represents a candidate Angle θ.

Update the velocity and position of the particles according to the standard PSO update formula.

Calculate the objective function value J(θ) corresponding to each particle.

Update the historical optimal solution and the global optimal solution for each particle.

3) Local search stage (Nelder-Mead) :

Taking the global optimal solution found by PSO as the initial point, a fine search is conducted.

locally using the Nelder-Mead method to obtain the final optimal solution.

4) Output result:

Including the optimal force application Angle and the corresponding least effort.

For a horizontal ground (with an inclination Angle of zero), according to theoretical calculations, the optimal force application Angle is approximately 11.31 degrees, and the most effortless push-pull force is 38.4 Newtons. The simulation was carried out using the hybrid optimization algorithm, and the obtained results were very close to the theoretical values. The force application Angle was 11.3 degrees, and the minimum push-pull force was 38.4 Newtons. Under the condition of an inclined ground (with an inclination Angle of 15 degrees), theoretical calculations also show that the optimal force application Angle is 11.31 degrees, and the most effortless push-pull force is 86.8 Newtons. The simulation results show that the force application Angle obtained through the optimization algorithm is 11.3 degrees, and the minimum push-pull force is 86.9 Newtons.

4.3. Question 3 Model Establishment and Solution

4.3.1. Model Establishment

First, conduct a decomposition analysis of the geometric relationship and force. Let the length of the pull rod be L. Due to the geometric relationship between the handheld position and the connection position of the suitcase, the horizontal and vertical force arms are respectively:

$$x = L\cos\theta, h = L\sin\theta \tag{14}$$

The suitcase is subjected to the force of gravity. When a force at a fixed Angle is applied, this force can be decomposed into components in the horizontal and vertical directions. The height of the force application point will change the torque balance and thereby affect the frictional force that needs to be overcome. Therefore, a friction correction term related to the force distribution needs to be introduced. We proposed an empirical model to represent the force required during pushing and pulling. Therefore, the specific model formula obtained is as follows:

$$F_x = F\cos\delta, F_y = F\sin\delta.$$
$$F(L) = \frac{A}{L+B} + C\cdot L + D \tag{15}$$

This model consists of two parts: One part has a linear relationship with the length of the push-pull rod, reflecting that when the push-pull rod is shorter, due to the lower force application point, the friction force is greater, and thus a greater push-pull force is required. Another part increases linearly with the increase in the length of the push-pull rod, indicating that an overly long rod may bring additional difficulty due to a decline in the body's output capacity or discomfort in arm extension. Overall, this model integrates the influences of various parameters in the system, such as mass, coefficient of friction, and geometric dimensions, etc.

$$mg_x = mg\sin\gamma, mg_y = mg\cos\gamma.$$
$$F(L) = \frac{A^{'}}{L+B^{'}} + C^{'}\cdot L + D^{'} \tag{16}$$

Among them, parameters A',B',C', and D' are related to γ and other system parameters. Next, we will conduct an analysis of force balance and the influence of friction. Taking the vertical ground as an example, the force analysis of the suitcase can list the balance conditions:

$$\begin{aligned} &N = mg - F\sin\delta + \Delta N(L), \\ &f_r = \mu N \\ &F\cos\delta \geq \mu[mg - F\sin\delta + \Delta N(L)] \end{aligned} \quad (17)$$

Among them, $\Delta N(L)$ takes into account the correction of the normal force distribution due to the change in the height of the application point. By using this inequality, the relationship between F and L can be inversely deduced. Similarly, the corresponding relationship can also be derived for inclined ground (note that at this time, the relationship between the gravitational component and the normal force will introduce the influence of γ). Next, parameter fitting is carried out using the nonlinear least square method. Since the constants in the above formula are difficult to be directly analyzed and determined, a set of {Li,Fi} data can be obtained through experimental data, and then the model parameters are fitted using the nonlinear least square method. Taking the vertical ground as an example, the objective function is constructed as follows:

$$\min_{A,B,C,D} \sum_{i=1}^{n} [F_i - (\frac{A}{L_i + B} + C \cdot L_i + D)]^2 \quad (18)$$

To further solve the optimization problem, after obtaining the model expression, it is further necessary to determine the "least laborious" optimal pull rod length Lopt and the corresponding minimum push-pull force Fmin.

$$L_{opt} = \arg\min_{L} F(L) \quad (19)$$

$$\begin{aligned} &\left.\frac{dF(L)}{dL}\right|_{L=L_{opt}} = 0, \left.\frac{d^2F(L)}{dL^2}\right|_{L=L_{opt}} > 0, \\ &F_{\min} = F(L_{opt}) = \frac{A}{\sqrt{\frac{A}{C}}} + C\left(\sqrt{\frac{A}{C}} - B\right) + D. \end{aligned} \quad (20)$$

4.3.2. Model Solution and Result Analysis

Since the model function may be a non-convex function, in order to obtain the global optimal solution, this paper adopts the simulated annealing algorithm (SA) combined with the local search strategy.

Through numerical simulation experiments, it can be known that a series of push-pull force F data under different pull rod lengths L were measured in the experiment. Suppose that for the horizontal ground situation, the parameters are obtained through nonlinear least squares fitting

Both experiments and theories show that within a certain range, increasing the length L of the pull rod can significantly reduce the force F required for pushing and pulling (the reason is that after the force application point is raised, the compressive force on the wheel is partially reduced, thereby reducing the friction force). However, when L exceeds a certain critical value, due to the inconvenience of human force application or mechanical structure limitations, F will instead rise, presenting a U-shaped characteristic.

Among them, parameter A (related to friction force) and C have a greater influence on L. Sensitivity analysis shows that if the static friction coefficient or m increases, the mechanical advantage needs to be compensated by a longer pull rod. However, if C is relatively large (indicating a sharp decline in the efficiency of human force application), the optimal length of the pull rod will be correspondingly shortened.

4.4. Question 4: Model Establishment and Solution

4.4.1. Model Establishment

First, mechanical analysis and force balance analysis should also be conducted. To ensure that the suitcase can overcome gravity and friction on an inclined road surface, it is necessary to analyze the relationship between the applied force and the direction of the pull rod. The Angle between the applied force and the horizontal direction is a certain degree, and the relative relationship between this force and the inclined plane is also taken into consideration. The components of this force can be decomposed into the components along the inclined plane and the components perpendicular to the inclined plane. The component along the inclined plane can be expressed as the difference between the force applied and the Angle of the pull rod, while the component perpendicular to the inclined plane is related to the sine value of the Angle. The following mechanical decomposition model is obtained, with specific formulas:

$$\begin{gathered} F_{\parallel} = F\cos(\theta - \alpha^{'}), F_{\perp} = F\sin(\theta - \alpha^{'}) \\ F\cos(\theta - \alpha^{'}) \geq mg\sin\alpha + \mu(mg\cos\alpha - F\sin(\theta - \alpha^{'})) \\ F \geq \frac{mg\sin\alpha + \mu mg\cos\alpha}{\cos(\theta - \alpha^{'}) + \mu\sin(\theta - \alpha^{'})} \end{gathered} \tag{21}$$

To overcome the gravitational component and frictional force on the inclined plane, and to take into account the influence of the applied force on the normal pressure, it is required that the force component along the inclined plane direction be greater than or equal to the sum of the gravitational component and the frictional force component. After sorting, the expression of the minimum applied force obtained is related to the Angle of the inclined plane, the coefficient of friction and the applied Angle. Among them, the Angle at which the force is applied and the geometric relationship between the pull rod and the inclined plane are closely related. For instance, if there is a certain height difference between the handle and the center of gravity of the suitcase, the normal force can be reduced by adjusting the Angle, thereby lowering the friction. This indicates that in practical applications, by rationally designing the Angle of the pull rod and the direction of force application, the force required to push and pull the suitcase can be effectively reduced.

The minimum force and direction applied when the length of the pull rod remains unchanged can be expressed by the following formulas respectively:

$$F_{\min}(\theta)=\frac{mg\sin\alpha+\mu mg\cos\alpha}{\cos(\theta-\alpha^{'})+\mu\sin(\theta-\alpha^{'})}$$
$$\min_{\theta\in[\theta_{\min},\theta_{\max}]} F_{\min}(\theta) \quad (22)$$

When the length of the pull rod is variable, the change in the length L of the pull rod will affect the position of the force application point, thereby influencing the balance of the normal pressure and moment on the suitcase. After considering the positional change, the new positive pressure expression is:

$$N=mg\cos\alpha-F\sin(\theta-\alpha^{'})+\Delta N(L) \quad (23)$$

Among them, $\Delta N(L)$ reflects the leverage effect caused by the change in the length of the pull rod (for example, by changing the height of the force application point, part of the gravity is decomposed and compensated). Therefore, the objective function can be written as:

$$F_{\min}(\theta,L)=\frac{mg\sin\alpha+\mu[mg\cos\alpha-F\sin(\theta-\alpha^{'})+\Delta N(L)]}{\cos(\theta-\alpha^{'})+\mu\sin(\theta-\alpha^{'})} \quad (24)$$

The problem is transformed into solving it under the given $\Delta N(L)$ model:

$$\min_{\theta\in[\theta_{\min},\theta_{\max}]} F_{\min}(\theta,L)$$
$$\Delta N(L)=k(L-L_0) \quad (25)$$

The specific $\Delta N(L)$ form should be established based on the actual geometric parameters. For instance, if the height difference between the front edge of the suitcase and the center of gravity is set to h, the normal pressure can be reduced when the pull rod is extended.

4.4.2. Model Solution and Result Analysis

Due to the nonlinear and continuous characteristics of the objective function, in order to improve the solution efficiency and global search ability, the following collaborative optimization process is introduced in this paper:

The first step is Latin hypercube sampling, which involves uniform sampling within the design space to obtain sufficient discrete sample points, providing data support for the subsequent construction of the response surface. Secondly, a response surface regression analysis is conducted. An approximate response surface model of the objective function (such as a polynomial regression model) is established using the sampled data to reduce the computational load in the subsequent solution process. Finally, the genetic algorithm is selected for optimization and solution. A global search is conducted on the response surface model to find the optimal combination of design variables that minimizes F.

The final result visualization diagram and specific result table can be obtained through simulation experiments, as shown below

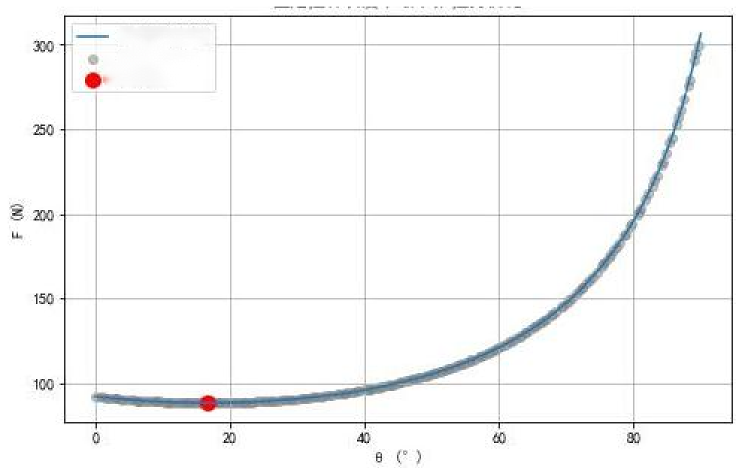

Figure 3. The optimization result of the minimum push-pull force under a fixed pull rod length can be viewed

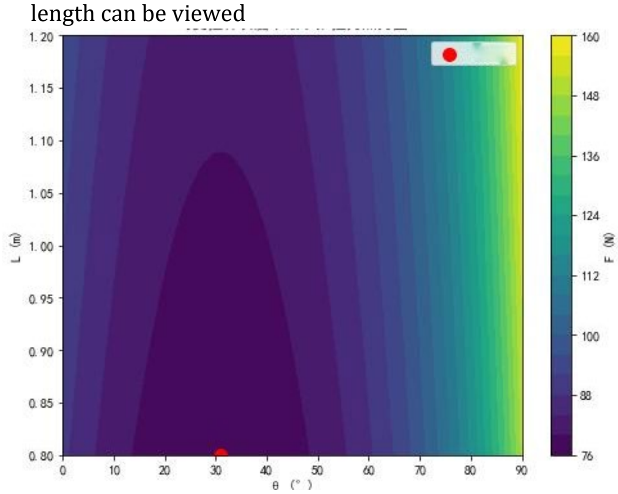

Figure 4. Minimum tension heat map under variable pull rod length

For the case where the length of the pull rod is fixed, only the force application Angle needs to be adjusted. For the variable pull rod length, there is greater room for optimization, which can further reduce the push-pull force. This multi-parameter optimization model utilizes the method of LHS sampling, response surface construction and genetic algorithm collaborative optimization, which not only takes into account the ability of global search, but also ensures the computational efficiency. In the simulation experiment, the influence of different parameter combinations on push-pull force was visually demonstrated through three-dimensional surface diagrams and heat maps, providing strong decision support for actual engineering design.

5. Evaluation, Improvement and Promotion of the Model

5.1. Advantages of the Model

High accuracy: By constructing a two-dimensional force analysis model and introducing the vector decomposition method based on Newtonian mechanics principles, the force conditions of the suitcase under different operation modes can be accurately analyzed, providing a solid theoretical foundation for the optimization of pushing and pulling forces. Meanwhile, the total force is expressed by a nonlinear function and fitted in combination with experimental data, ensuring the high precision of the model under complex working conditions. In addition, the introduction of sensitivity analysis further quantifies the impact of parameter changes on push-pull force, enabling the model to precisely capture key factors and provide accurate guidance for optimization strategies.

Strong adaptability: This model can flexibly handle various complex scenarios. Under different working conditions such as horizontal roads, inclined roads, ground with different friction coefficients, and variations in the length of the pull rods, the optimal push-pull strategy can be found through corresponding optimization models and algorithms. For instance, in Problem Four, by constructing a multi-dimensional optimization model that includes three variables: ground tilt Angle, tie rod Angle, and tie rod length, it is possible to effectively address multi-parameter optimization problems under inclined road surfaces. This powerful adaptability enables the model to play a significant role in various practical application scenarios.

Wide universality: The core optimization algorithms of this model (such as the improved particle swarm optimization algorithm, multi-objective optimization framework, simulated annealing algorithm, etc.) have wide universality. These algorithms are not only applicable to the optimization problem of the pushing and pulling force of suitcases, but can also be appropriately modified and applied to other scenarios that require optimization. For instance, in the design of portable devices, optimize the structural parameters of the equipment to reduce operational force; in the logistics and warehousing field, optimize the layout of shelves and the design of handling equipment; in industrial automation, optimize the path planning of robots and the operation of mechanical arms, etc. By adjusting the input parameters and optimization objectives of the model, it can quickly adapt to different optimization requirements and provide efficient solutions for various engineering problems.

5.2. Disadvantages of the Model

The model assumes that the shape of the suitcase is regular, the center of gravity distribution is uniform, and the friction coefficient of the ground is uniform within a certain range, which may lead to errors in the practical application of the model.

Complex models enhance the accuracy and adaptability of the results, but they also lead to longer solution times, higher computational costs, and may not be suitable for some application scenarios with high real-time requirements. At the same time, they increase the difficulty of model debugging and verification.

5.3. Model Promotion

The vector decomposition model, nonlinear optimization model and multi-dimensional optimization model established in this research can be further extended to a wider range of application scenarios. For instance, in the field of industrial logistics, this model can be used to reduce energy consumption and enhance handling efficiency. In the field of warehousing, it can be used to optimize the structure and layout of shelves, improve the utilization rate of warehousing space and the efficiency of goods entry and exit. In the field of robotics, it can be used to analyze the movement trajectory, joint angles and forces of robots. In addition, the algorithmic foundation of the model - nonlinear programming, due to its ability to handle complex working conditions, can also be applied to other types of optimization problems, such as aircraft structure optimization and energy system optimization. By adjusting the algorithm parameters and objective function, the model can adapt to different problem scenarios and data characteristics, achieving a wider range of applications.

References

[1] Zhu Yingjie, Guo Limin, & Chen Dong. (2011). College Physics (Second Edition) Beijing: Higher Education Press.

[2] Chen Xiru. (2009). Engineering Mechanics (Statics Part). Beijing: Tsinghua University Press.

[3] Wang Zhongxiao, & Zhou Guodong. (2013). Fundamentals of Tribology. Beijing: Machinery Industry Press.

[4] Wang Jue. (2009). Response Surface Method and Experimental Design Beijing: Science Press.

[5] Sun Jiadong, Wu Jianping, & Liu Hongbo. (2004). Genetic Algorithm and Its Application in Engineering Optimization. Beijing: Machinery Industry Press.

[6] Zhang Yong, & Li Xiaoming. (2009). Multi-objective Optimization Method and Its Application Beijing: Science Press.

[7] Liu Min. (2010). Computer Experiment Design Methods Beijing: Higher Education Press.

[8] Wang Tao. (2011). Modern Experimental Design and Optimization Methods Beijing: Machinery Industry Press.